LAKSHYA

MHTCET 2025

Mathematics | Lecture - 01

Differentiation

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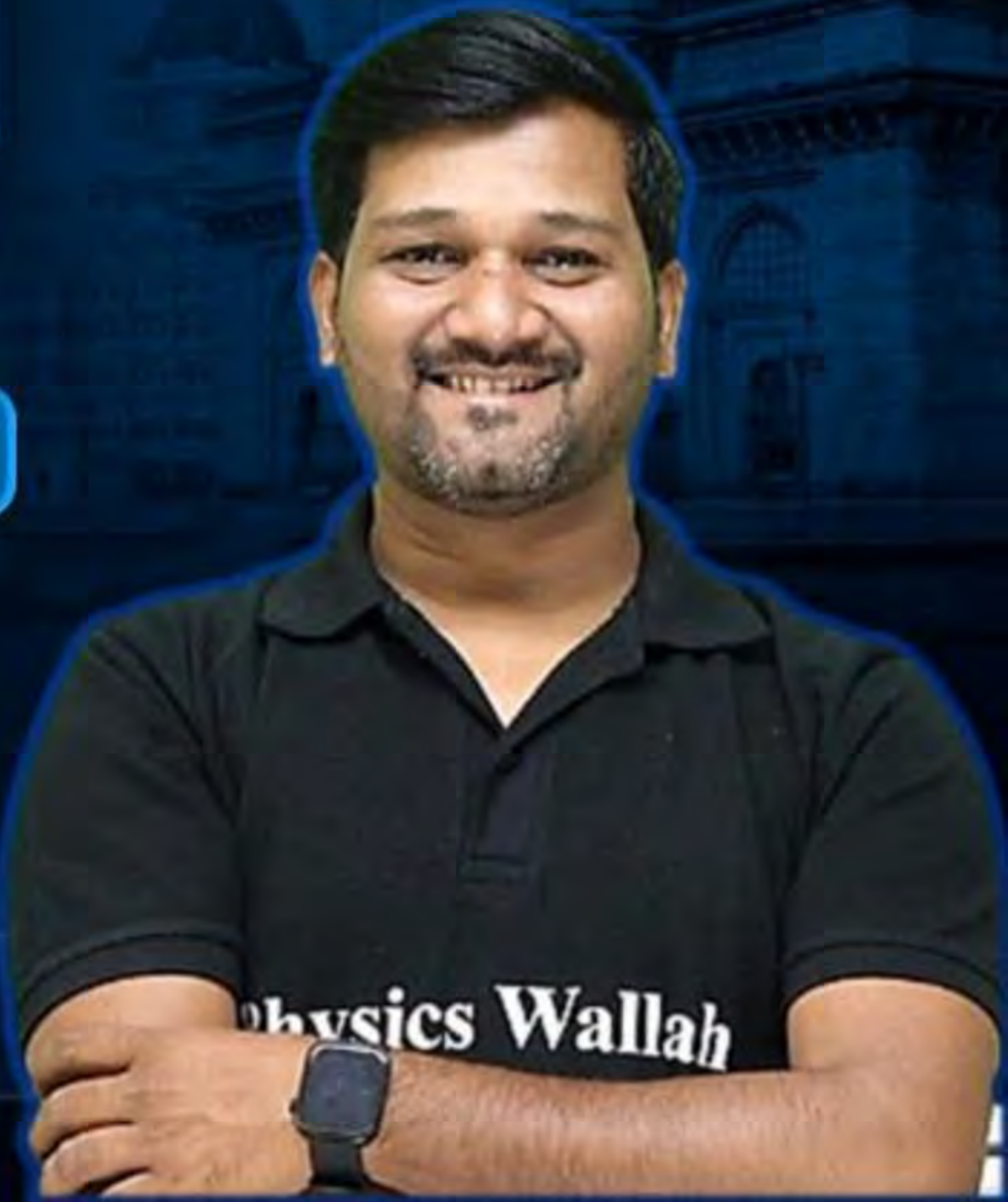


Recap *of previous lecture*

1. Logarithm ✓

Topics *to be covered*

1. Derivatives of standard functions
2. Derivatives of composite function
3. Practice Questions

Basics of differentiation

$\frac{ds}{dt} \rightsquigarrow$ velocity $\qquad \frac{dv}{dt} \rightsquigarrow$ acceleration

$$y = f(x)$$

$$\frac{dy}{dx} \qquad f'(x)$$

first principle of derivative

$$\boxed{f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}}$$

Derivatives of some standard Function

1. $\frac{d}{dx}C = 0$

Example:-

i) $\frac{d}{dx}10 = 0$

ii) $\frac{d}{dx}(-5) = 0$

iii) $\frac{d}{dx}(1000) = 0$

2. $\frac{d}{dx}x^n = nx^{n-1}$

Example:-

i) $\frac{d}{dx}x^7 = 7x^{7-1}$
$= 7x^6$

ii) $\frac{d}{dx}2x^2 = 2\frac{d}{dx}x^2$
$= 2(2x^1)$
$= 2(2x)$
$= 4x$

$y = f(x)$	$\frac{dy}{dx} = f'(x)$
c (Constant)	0
x^n	nx^{n-1}
$\frac{1}{x}$	$-\frac{1}{x^2}$
$\frac{1}{x^n}$	$-\frac{n}{x^{n+1}}$
$\sqrt{x}$	$\frac{1}{2\sqrt{x}}$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\sec^2 x$

Derivatives of some standard Function

(3) $\boxed{\frac{d}{dx}\left(\frac{1}{x}\right) = \frac{-1}{x^2}}$

Example:

$$\frac{d}{dx}\left(\frac{1}{x}\right) = \frac{d}{dx}(x^{-1})$$
$$= -1\,x^{-1-1}$$
$$= -1\,x^{-2}$$
$$= \frac{-1}{x^2}$$

(4) $\frac{d}{dx}\left(\frac{1}{x^n}\right) = -\frac{n}{x^{n+1}}$

Example:-

$$\frac{d}{dx}\left(\frac{1}{x^n}\right) = \frac{d}{dx}x^{-n}$$
$$= -n\,x^{-n-1}$$
$$= -n\,x^{-(n+1)}$$
$$= \frac{-n}{x^{n+1}}$$

$y = f(x)$	$\frac{dy}{dx} = f'(x)$
c (Constant)	0
x^n	nx^{n-1}
$\frac{1}{x}$	$-\frac{1}{x^2}$
$\frac{1}{x^n}$	$-\frac{n}{x^{n+1}}$
$\sqrt{x}$	$\frac{1}{2\sqrt{x}}$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\sec^2 x$

Derivatives of some standard Function

(5) $\frac{d}{dx}\sqrt{x} = \frac{1}{2\sqrt{x}}$

Example:

$$\frac{d}{dx} x^{\frac{1}{2}} = \frac{1}{2} x^{\frac{1}{2}-1}$$
$$= \frac{1}{2} x^{-\frac{1}{2}}$$
$$= \frac{1}{2} \cdot \frac{1}{x^{\frac{1}{2}}}$$
$$= \frac{1}{2\sqrt{x}}$$

(6) $\frac{d}{dx}\sin x = \cos x$

(7) $\frac{d}{dx}(\cos x) = -\sin x$

(8) $\frac{d}{dx}(\tan x) = \sec^2 x$

(9) $\frac{d}{dx}(\operatorname{cosec} x)$
$= -\operatorname{cosec} x \cot x$

(10) $\frac{d}{dx}(\sec x) = \sec x \tan x$

(11) $\frac{d}{dx}(\cot x) = -\operatorname{cosec}^2 x$

$y = f(x)$	$\frac{dy}{dx} = f'(x)$
c (Constant)	0
x^n	nx^{n-1}
$\frac{1}{x}$	$-\frac{1}{x^2}$
$\frac{1}{x^n}$	$-\frac{n}{x^{n+1}}$
$\sqrt{x}$	$\frac{1}{2\sqrt{x}}$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\sec^2 x$

Derivatives of some standard Function

(12) $\frac{d}{dx}e^x = e^x$

(13) $\frac{d}{dx}a^x = a^x \log a$

$\frac{d}{dx}\left(\frac{1}{2}\right) = 0$

$\frac{d}{dx}(\sqrt{3}) = 0$

Example :-

① $\frac{d}{dx}5^x = 5^x \log 5$

② $\frac{d}{dx}3^x = ?$

$y = f(x)$	$\frac{dy}{dx} = f'(x)$
✓ $\sec x$	$\sec x \tan x$
✓ $\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$
✓ $\cot x$	$-\operatorname{cosec}^2 x$
e^x	e^x
a^x	$a^x \log a$
$\log x$	$\frac{1}{x}$
$\log_a x$	$\frac{1}{x \log a}$

Derivatives of some standard Function

(14) $\frac{d}{dx}\log x = \frac{1}{x}$

(15) $\frac{d}{dx}\log_a x = \frac{1}{x\log a}$

<u>Example</u>

$\frac{d}{dx}\left[\frac{\log x}{\log a}\right]$

$= \frac{1}{\log a}\frac{d}{dx}\log x$

$= \frac{1}{\log a}\frac{1}{x}$

$= \frac{1}{x\log a}$

$y=f(x)$	$\frac{dy}{dx}=f'(x)$
$\sec x$	$\sec x \tan x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$
$\cot x$	$-\operatorname{cosec}^2 x$
e^x	e^x
a^x	$a^x \log a$
$\log x$	$\frac{1}{x}$
$\log_a x$	$\frac{1}{x\log a}$

Rules of Differentiation

If u and v are differentiable functions of x such that

(i) $y = u \pm v$ then $\frac{dy}{dx} = \frac{du}{dx} \pm \frac{dv}{dx}$

Example:

$$y = x^2 + e^x$$

diff w.r.t x

$$\frac{dy}{dx} = \frac{d}{dx}x^2 + \frac{d}{dx}e^x$$

$$= 2x + e^x$$

Rules of Differentiation

If u and v are differentiable functions of x such that

(ii) $y = uv$ then $\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$

$$\boxed{\frac{d}{dx}(\mathrm{I} \times \mathrm{II}) = \mathrm{I}\frac{d}{dx}\mathrm{II} + \mathrm{II}\frac{d}{dx}\mathrm{I}}$$

Example:-

$y = xe^x$

diff. w.r.t x

$\frac{dy}{dx} = x\frac{d}{dx}e^x + e^x\frac{d}{dx}x$

$= xe^x + e^x \cdot 1$

$= xe^x + e^x$

$= e^x(x+1)$

$\frac{d}{dx}x = \frac{d}{dx}x^1$

$= 1x^{1-1}$

$= 1x^0$

$= 1$

Rules of Differentiation

If u and v are differentiable functions of x such that

(iii) $y = \frac{u}{v}$ where $v \neq 0$ then $\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

$$\frac{d}{dx}\left(\frac{I}{II}\right) = \frac{II\frac{d}{dx}I - I\frac{d}{dx}II}{(II)^2}$$

Example:

i) $y = \frac{\sin x}{x}$

diff w.r.t x

$$\frac{dy}{dx} = \frac{x\frac{d}{dx}\sin x - \sin x\frac{d}{dx}x}{(x)^2}$$

$$= \frac{x\cos x - \sin x}{x^2}$$

Composite Function

$f[g(x)] = fog$ $\quad$ $g[f(x)] = gof$

$y = f(u)$, $u = g(x) \Rightarrow y = f[g(x)]$

$y \to u \to x$ $\quad$ $\dfrac{dy}{dx} = \dfrac{dy}{du} \times \dfrac{du}{dx}$

chain Rule

$y \to u \to w \to z \to x$ $\quad$ $\boxed{\dfrac{dy}{dx} = \dfrac{dy}{du} \times \dfrac{du}{dw} \times \dfrac{dw}{dz} \times \dfrac{dz}{dx} \cdots}$

Derivatives of Composite Function

$$\frac{d}{dx} f[g(x)]$$

$$= f'[g(x)] \frac{d}{dx} g(x)$$

$$= f'[g(x)] \, g'(x)$$

Question

Differentiate w.r.t. x.

(i) $(x^3 - 2x - 1)^5$

Let, $y = (x^3 - 2x - 1)^5$

diff w.r.t x

$\frac{dy}{dx} = \frac{d}{dx}[x^3 - 2x - 1]^5$

$= 5[x^3 - 2x - 1]^4 \frac{d}{dx}(x^3 - 2x - 1)$

$= 5[x^3 - 2x - 1]^4 (3x^2 - 2)$

(ii) $\left(2x^{\frac{3}{2}} - 3x^{\frac{4}{3}} - 5\right)^{\frac{5}{2}}$ $\quad \frac{5}{2} - \frac{1}{1} = \frac{3}{2}$

Let $y = \left(2x^{\frac{3}{2}} - 3x^{\frac{4}{3}} - 5\right)^{\frac{5}{2}}$

diff w.r.t x

$\frac{dy}{dx} = \frac{5}{2}\left[2x^{\frac{3}{2}} - 3x^{\frac{4}{3}} - 5\right]^{\boxed{\frac{5}{2}-1}} \frac{d}{dx}\left[2x^{\frac{3}{2}} - 3x^{\frac{4}{3}} - 5\right]$

$= \frac{5}{2}\left[2x^{\frac{3}{2}} - 3x^{\frac{4}{3}} - 5\right]^{\frac{3}{2}} \left[\cancel{2} \times \frac{3}{\cancel{2}} x^{\frac{3}{2}-1} - \cancel{3} \times \frac{4}{\cancel{3}} x^{\frac{4}{3}-1}\right]$

$= \frac{5}{2}\left[2x^{\frac{3}{2}} - 3x^{\frac{4}{3}} - 5\right]^{\frac{3}{2}} \left[3x^{\frac{1}{2}} - 4x^{\frac{1}{3}}\right]$

Question

Differentiate w.r.t. x.

(iv) $\sqrt{x^2+\sqrt{x^2+1}}$

$$y=\left[x^2+\sqrt{x^2+1}\right]^{\frac{1}{2}}$$

$$\frac{d}{dx}\sqrt{x}=\frac{1}{2\sqrt{x}}$$

diff w.r.t x

$$\frac{dy}{dx}=\frac{1}{2\sqrt{x^2+\sqrt{x^2+1}}}\frac{d}{dx}\left[x^2+\sqrt{x^2+1}\right]$$

$$=\downarrow\left[2x+\frac{1}{2\sqrt{x^2+1}}\frac{d}{dx}(x^2+1)\right]$$

$$=\frac{1}{2\sqrt{x^2+\sqrt{x^2+1}}}\left[2x+\frac{1}{\cancel{2}\sqrt{x^2+1}}(\cancel{2}x)\right]$$

$$\rightarrow\frac{1}{2\sqrt{x^2+\sqrt{x^2+1}}}\left[2x+\frac{x}{\sqrt{x^2+1}}\right]$$

Question

Differentiate w.r.t. x.

(v) $\frac{3}{5\sqrt[3]{(2x^2-7x-5)^5}}$

$5 \times \frac{1}{3} = \frac{5}{3}$

$y = \frac{3}{5} \frac{1}{\left[(2x^2-7x-5)^5\right]^{\frac{1}{3}}}$

$= \frac{3}{5} \frac{1}{\left[2x^2-7x-5\right]^{\frac{5}{3}}}$

$= \frac{3}{5} (2x^2-7x-5)^{-\frac{5}{3}}$

Question

A table of values of f, g, f' and g' is given

(i) If $r(x) = f[g(x)]$ find $r'(2)$.

(ii) If $R(x) = g[3 + f(x)]$ find $R'(4)$.

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
2	1	6	-3	4
4	3	4	5	-6
6	5	2	-4	7

Sol: i) $r(x) = f[g(x)]$

diff w.r.t. x

$r'(x) = f'[g(x)] \frac{d}{dx} g(x)$

$r'(x) = f'[g(x)]\, g'(x)$

$r'(2) = f'[g(2)]\, g'(2)$

$= f'[6]\, 4$

$= -4 \times 4$

$= -16$

ii) $R(x) = g[3 + f(x)]$

$R'(x) = g'[3 + f(x)] \frac{d}{dx}[3 + f(x)]$

$= g'[3 + f(x)]\, f'(x)$

$R'(4) = g'[3 + f(4)]\, f'(4)$

$= g'[3+3]\, 5$

$g'(6) \cdot 5$

$7 \times 5 \Rightarrow 35$

Question

If $h(x) = \sqrt{4f(x) + 3g(x)}, f(1) = 4, g(1) = 3, f'(1) = 3, g'(1) = 4$ find $h'(1)$.

$$h'(x) = \frac{1}{2\sqrt{4f(x)+3g(x)}} \frac{d}{dx}[4f(x)+3g(x)]$$

$$h'(x) = \frac{1}{2\sqrt{4f(x)+3g(x)}}[4f'(x)+3g'(x)]$$

$$h'(1) = \frac{1}{2\sqrt{4f(1)+3g(1)}}[4f'(1)+3g'(1)]$$

$$= \frac{1}{2\sqrt{4\times4+3\times3}}[4\times3+3\times4]$$

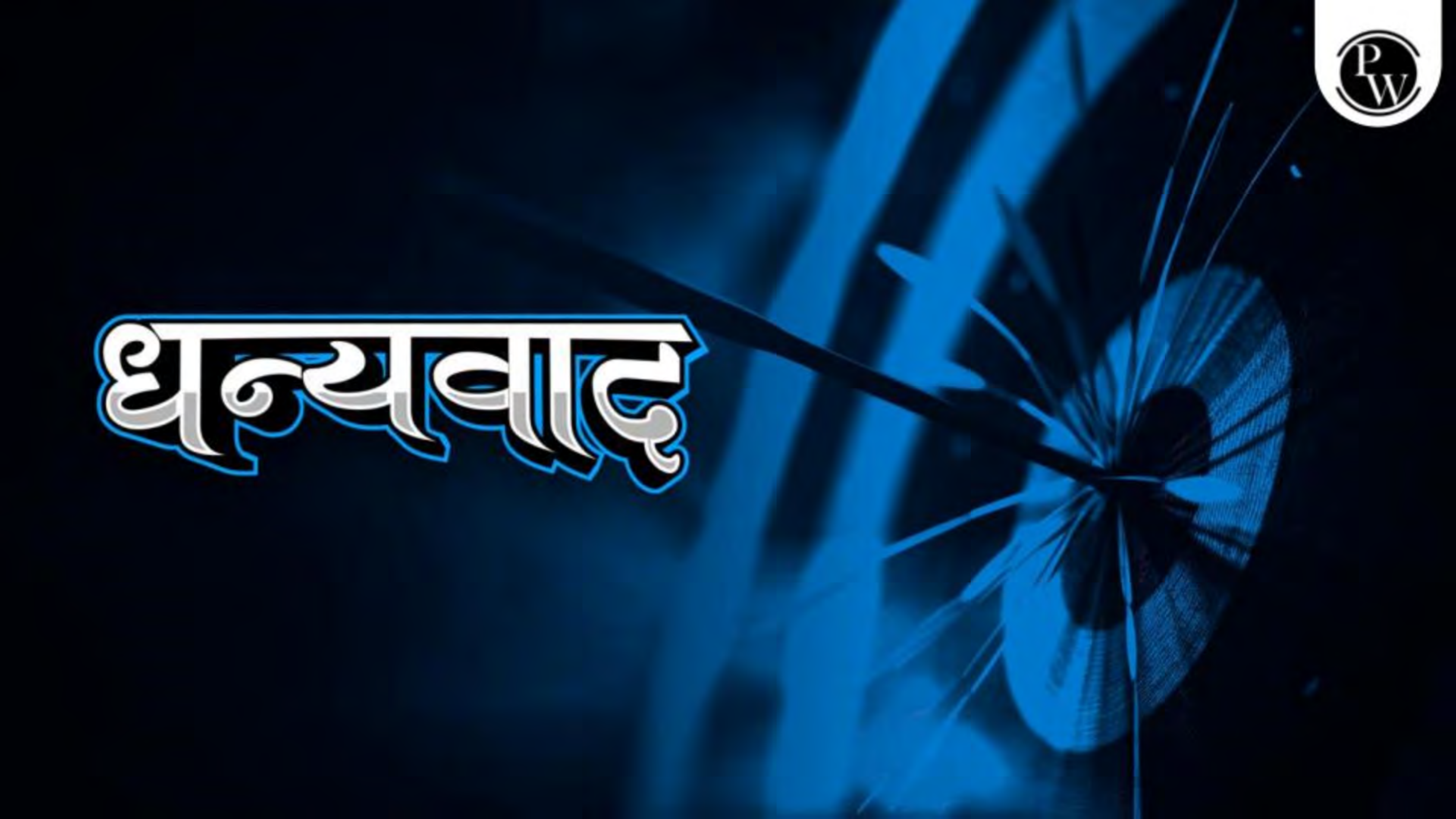
$$= \frac{1}{2\sqrt{16+9}}[12+12]$$

$$\frac{1}{\cancel{2}\sqrt{25}}(\cancel{24}^{12})$$

$$= \frac{12}{5}$$

Homework

Revise all formulae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